math 251 - week 14 - ch10 - Linear Programming Problem (LPP)

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def LPP: Maximizing or Minimizing a Linear function under some Linear Constraints, is called LPP

Mathematical formulation of LPP:

Max/Min $Z = C'\bar{x}$ → objective function.

Subject to $A\bar{x} \leqslant b$ → constraints.

$\bar{x} \geqslant 0$ → non-negative.

* We can solve LPP in <u>3 methods</u>:

$\boxed{1}$ Graphical Method.

$\boxed{2}$ Simplex Method.

$\boxed{3}$ Dual simplex Method.

Ex) Using a graphical method to Solve LPP

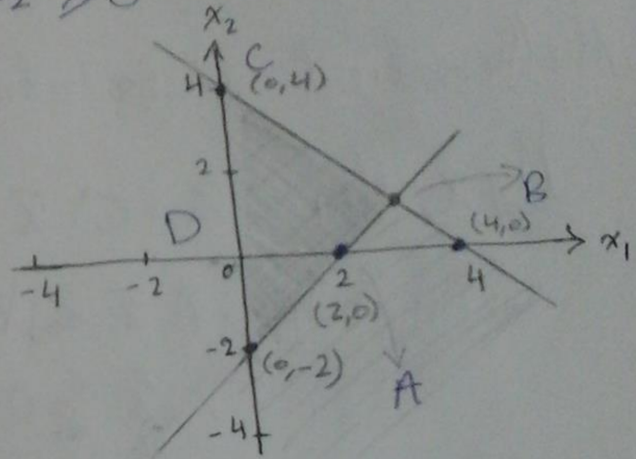
Max $\quad Z = 3x_1 + 2x_2 \quad$ —— ①

subject to $\quad x_1 + x_2 \leqslant 4 \quad$ —— ②

$\qquad\qquad x_1 - x_2 \leqslant 2 \quad$ —— ③

$\qquad\qquad x_1 , x_2 \geqslant 0$

Sol.



for equation ② :

$x_1 + x_2 = 4$

x_1	0	4
x_2	4	0

$\Rightarrow (x_1, x_2) = (0, 4) \rightarrow 1^{st}$ Point

$\qquad\qquad\quad = (4, 0) \rightarrow 2^{nd}$ Point

Since $0 + 0 \leqslant 4$ from ②

$0 \leqslant 4 \Rightarrow$ True Statement.

feasible Reagion Lie on the origion

اتجاه-نحو toward to the Origion Point.

From equation (3)

$x_1 - x_2 = 2$

x_1	0	2
x_2	-2	0

$\Rightarrow (x_1, x_2) = (0, -2) \rightarrow 1^{st}$ Point

$= (2, 0) \rightarrow 2^{nd}$ Point

Since $0 - 0 \leq 2$

$0 \leq 2 \Rightarrow$ True

the feasible Reagion lie ~~on~~ toward the [redacted] Origin.

The common feasible Reagion is

D → (0,0) A → (2,0) B → ? C → (0,4)

$$\begin{aligned} x_1 + \cancel{x_2} &= 4 \\ x_1 - \cancel{x_2} &= 2 \\ \hline 2x_1 &= 6 \end{aligned}$$

$\frac{\cancel{2}x_1}{\cancel{2}} = \frac{6}{2} \Rightarrow \boxed{x_1 = 3}$

$x_2 = 4 - x_1 = 4 - 3 \Rightarrow \boxed{x_2 = 1}$

$\rightarrow (x_1, x_2) = (3, 1)$

Draw the table:

Serial No.	Corner Point	$Z = 3x_1 + 2x_2$
1	A (2, 0)	$Z = 3(2) + 2(0) = 6$
2	B (3, 1)	$Z = 3(3) + 2(1) = 11$
3	C (0, 4)	$Z = 3(0) + 2(4) = 8$
4	D (0, 0)	$Z = 3(0) + 2(0) = 0$

Maximum value is 11 $\Rightarrow$ B point

∴ The Maximum value will be at the Point B(3, 1)

hence the optimal solution is $x_1 = 3$

$x_2 = 1$

Note that the Minimum value will be at the Point D (0, 0)

Using Graphical

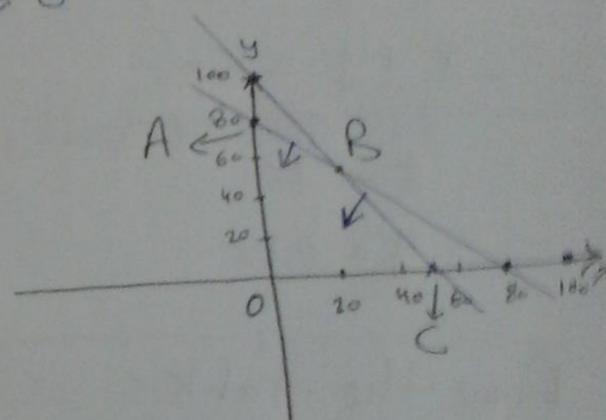
Max $Z = 50x_1 + 18x_2$ —— ①

subject to $2x + y \leq 100$ —— ②

$x + y \leq 80$ —— ③

$x, y \geq 0$

sol.



from equation ②:

$2x + y = 100$

x	0	50
y	100	0

$0 + 0 \leq 100$ True

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from equation ③:

$x + y = 80$

x	0	80
y	80	0

$0 + 0 \leq 80$

True

The Corner points are

$A \rightarrow (0,80)$ $\quad O \rightarrow (0,0)$ $\quad C \rightarrow (50,0)$ $\quad B \rightarrow ??$

$$
\begin{array}{r}
2x + y = 100 \\
- x \pm y = \underset{-}{8}0 \\
\hline
x = 20 \\
y = 60
\end{array}
$$

Draw the table

S. No	Corner points	$Z = 50\,x_1 + 18\,x_2$
1	A $(0,80)$	$Z = 50(0) + 18(80) = 1440$
2	O $(0,0)$	$Z = 50(0) + 18(0) = 0$
3	C $(50,0)$	$Z = 50(50) + 18(0) = 2500$
4	B $(20,60)$	$Z = 50(20) + 18(60) = 2080$

The Maximum value will be at the Point

C (50,0)

The Minmum value will be at the Point

O (0,0)

Simplex Method

Ex] Use the simplex method to solve LPP

$$\text{Max } Z = 3x_1 + 2x_2$$

Subject to
$$x_1 + x_2 \leq 4$$
$$x_1 - x_2 \leq 2$$
$$x_1, x_2 \geq 0$$

Sol. 4 steps are there:-

1 step.1 Write the Standard form

$$\text{Max } Z = 3x_1 + 2x_2 + 0S_1 + 0S_2$$

Subject to
$$x_1 + x_2 + S_1 = 4$$
$$x_1 - x_2 + S_2 = 2$$
$$x_1, x_2, S_1, S_2 \geq 0$$

2) Step.2 We have 4 Varaibles and 2 equati[illegible]

So, $4-2=2$ variables equal to 0.

$x_1 = x_2 = 0$ So, $S_1 = 4$ $S_2 = 2$

$\overset{0}{x_1} + \overset{0}{x_2} + \overset{4}{S_1} = 4$

$\underset{0}{x_2} - \underset{0}{x_2} + S_1 = 2$

3) Step.3 Draw the table:

C_B	Basis	solution C_j	$\overset{3}{x_1}$	$\overset{2}{x_2}$	$\overset{0}{S_1}$	$\overset{0}{S_2}$	Ratio
0	S_1	4	1	1	1	0	4/1
0	S_2	2	Pivot element 1	-1	0	1	2/1
coefficient of S_1 and S_2	Z_j		0	0	0	0	
		$Z_j - C_j$	-3	-2	0	0	

Minimum positive Ratio

4) Step.4: check optimility

	x_1	x_2	s_1	s_2
1	0	1	1/2	-1/2
2 R_2+R_1	1	-1	0	1
R_3+5R_1	0	-5	0	3

2	0	2	1	-1	2/2
2	1	-1	0	1	2/-1
6	0	-5	0	3	

1	0	1	1/2	-1/2
3	1	0	1/2	1/2
	0	0	5/2	1/2

Since all the values are +ve

So the Solution is $x_1 = 3$, $x_2 = 1$

Max $Z = 11$